Algebra 2 – Things to Remember!

Exponents:
- \( x^0 = 1 \)
- \( x^{-m} = \frac{1}{x^m} \)
- \( x^m \cdot x^n = x^{m+n} \)
- \( x^{-m} = \frac{x^m}{x^n} \)
- \( x^{-n} = \frac{x^n}{x^m} \)
- \( (x^n)^m = x^{mn} \)
- \( \left( \frac{x}{y} \right)^n = \frac{x^n}{y^n} \)

Complex Numbers:
- \( \sqrt{-1} = i \)
- \( -1 = a + bi \) where \( a \geq 0 \)
- \( i^2 = -1 \)
- \( i^{14} = i^2 = -1 \) divide exponent by 4, use remainder, solve.
- \( (a + bi) \) conjugate \( (a - bi) \)
- \( (a + bi)(a - bi) = a^2 + b^2 \)
- \( |a + bi| = \sqrt{a^2 + b^2} \) absolute value=magnitude

Logarithms
- \( \log_y b = x \) \( \iff \) \( b = y^x \)
- \( \ln \log_e x = \) natural log
- \( \log_{10} \log_{10} x = \) common log
- Change of base formula:
  \( \log_b a = \frac{\log a}{\log b} \)

Factoring:
- Look to see if there is a GCF (greatest common factor) first. \( ab + ac = a(b + c) \)
- \( x^2 - a^2 = (x-a)(x+a) \)
- \( (x+a)^2 = x^2 + 2ax + a^2 \)
- \( (x-a)^2 = x^2 - 2ax + a^2 \)

Factor by Grouping:
- \( x^3 + 2x^2 - 3x - 6 \)
  \( (x^3 + 2x^2) - (3x + 6) \) group
- \( x^2(x+2) - 3(x+2) \) factor each
- \( (x^2 - 3)(x+2) \) factor

Variation:
- always involves the constant of proportionality, \( k \). Find \( k \) and then proceed.
  Direct variation: \( y = kx \)
  Inverse variation: \( y = \frac{k}{x} \)
  Varies jointly: \( y = kxj \)
  Combo: Sales vary directly with advertising and inversely with candy cost.

Exponentials
- \( e^x = \exp(x) \)
- \( b^x = b^y \) \( \iff \) \( x = y \) \( \) \( b > 0 \) and \( b \neq 1 \)
If the bases are the same, set the exponents equal and solve.

Solving exponential equations:
1. Isolate exponential expression.
2. Take \( \log \) or \( \ln \) of both sides.
3. Solve for the variable.

\( \ln(x) \) and \( e^x \) are inverse functions

\( \ln e^x = x \)
\( e^{\ln x} = x \)
\( e^{2\ln 3} = e^{\ln 9} = 9 \)

Logarithmic Equations
- \( ax^2 + bx + c = 0 \) (Set = 0.)
Solve by factoring, completing the square, quadratic formula.

Completing the square:
1. If other than one, divide by coefficient of \( x^2 \)
2. Move constant term to other side \( x^2 - 2x = 5 \)
3. Take half of coefficient of \( x \), square it, add to both sides \( x^2 - 2x + \left( \frac{1}{2} \right)^2 = 5 + \left( \frac{1}{2} \right)^2 \)
4. Factor perfect square on left side. \( (x - 1)^2 = 6 \)
5. Use square root property to solve and get two answers. \( x = 1 \pm \sqrt{6} \)

Sum of roots:
- \( r_1 + r_2 = -\frac{b}{a} \)

Product of roots:
- \( r_1 \cdot r_2 = \frac{c}{a} \)

Inequalities:
- \( x^2 + x - 12 \leq 0 \) Change to =, factor, locate critical points on number line, check each section.

\( (x+4)(x-3) = 0 \)
\( x = -4; \quad x = 3 \)

ANSWER: \(-4 \leq x \leq 3 \) or \([-4, 3]\) (in interval notation)

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Radicals: Remember to use fractional exponents.
\[ \sqrt{x} = x^{\frac{1}{2}}, \quad x^n = \sqrt[n]{x^n} = (\sqrt[n]{x})^n \]
\[ \sqrt[n]{a^n} = a \quad \text{and} \quad \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \]
\[ \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \]
Simplify: look for perfect powers.
\[ \sqrt{x^{12}} \cdot y^{17} = \sqrt{x^{12}} \cdot y^{16} = x^6 \cdot y^{\frac{16}{2}} \]
\[ \sqrt[3]{72x^9y^8z^7} = \sqrt[3]{8x^9y^6y^2z^3} = 2x^3y^2z\sqrt[3]{9y^2} \]
Rational Inequalities
\[ x^2 - 2x - 15 \geq 0 \]
The critical values from factoring the numerator are -3, 5.
The denominator is zero at \( x = 2 \).
Place on number line, and test sections.
\[ \begin{array}{ccc}
-3 & 0 & 2 & 5 \\
\end{array} \]
Equations: isolate the radical; square both sides to eliminate radical; combine; solve.
\[ 2x - 5\sqrt{x} - 3 = 0 \quad \rightarrow \quad (2x - 3)^2 = (5\sqrt{x})^2 \\
4x^2 - 12x + 9 = 25x \quad \rightarrow \quad \text{solve: } x = 9; \quad x = 1/4 \\
\]
CHECK ANSWERS. Answer only \( x = 9 \).

Working with Rationals (Fractions):
Simplify: remember to look for a factoring of -1:
\[ \frac{3x - 1}{1 - 3x} = \frac{-1(3x + 1)}{1 - 3x} = -1 \]
Add: Get the common denominator. Factor first if possible:
Multiply and Divide: Factor First

Solving Rational Equations:
Get rid of the denominators by mult. all terms by common denominator.
\[ \frac{22}{2x^2 - 9x - 5} - \frac{3}{2x + 1} = \frac{2}{x - 5} \]
multiply all by \( 2x^2 - 9x - 5 \) and get
\[ 22 - 3(x - 5) = 2(2x + 1) \]
\[ 22 - 3x + 15 = 4x + 2 \]
\[ 37 - 3x = 4x + 2 \]
\[ 35 = 7x \]
\[ 5 = x \]
Great! But the only problem is that \( x = 5 \) does not CHECK!!!! There is no solution. Extraneous root.
Motto: Always CHECK ANSWERS.

Equations of Circles:
\[ x^2 + y^2 = r^2 \quad \text{center origin} \]
\[ (x - h)^2 + (y - k)^2 = r^2 \quad \text{center at } (h,k) \]
\[ x^2 + y^2 + Cx + Dy + E = 0 \quad \text{general form} \]

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Complex Fractions:
Remember that the fraction bar means divide:
Method 1: Get common denominator top and bottom
\[ \frac{2}{x^2 - 4} \]
\[ \frac{4 - 2}{x} \]
\[ \frac{4}{x^2 - 4} \]
\[ \frac{x}{x^2} \]
\[ \frac{4 - 2}{x^2} \]
\[ \frac{x}{x^2} \]
\[ \frac{4 - 2}{x^2} \]
Method 2: Mult. all terms by common denominator for all.
\[ \frac{2}{x^2 - 4} \]
\[ \frac{4}{x^2 - 4} \]
\[ \frac{x}{x^2} \]
\[ \frac{4}{x^2} \]

Binomial Theorem:
\[ (a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \]

Functions: A function is a set of ordered pairs in which each \( x \)-element has only ONE \( y \)-element associated with it.

Vertical Line Test: is this graph a function?
Domain: \( x \)-values used; Range: \( y \)-values used
Onto: all elements in \( B \) used.
1-to-1: no element in \( B \) used more than once.
Composition: \( (f \circ g)(x) = f(g(x)) \)
Inverse functions \( f \& g \): \( f(g(x)) = g(f(x)) = x \)
Horizontal line test: will inverse be a function?

Transformations:
- \( f(x) \) over \( x \)-axis; \( f(-x) \) over \( y \)-axis
- \( f(x + a) \) horizontal shift; \( f(x) + a \) vertical shift
- \( f(ax) \) stretch horizontal; \( af(x) \) stretch vertical

Sequences
Arithmetic: \( a_n = a_1 + (n-1)d \)
\[ S_n = \frac{n(a_1 + a_n)}{2} \]
Geometric: \( a_n = a_1 \cdot r^{n-1} \)
\[ S_n = \frac{a_1(1 - r^n)}{1 - r} \]
Recursive: Example:
\[ a_1 = 4; \quad a_n = 2a_{n-1} \]

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### Trigonometry – Things to Remember!

#### Arc Length of a Circle
\[ \theta r \] (in radians)

#### Special Right Triangles
- **30°-60°-90° triangle**
  - side opposite 30° = \( \frac{1}{2} \) hypotenuse
  - side opposite 60° = \( \frac{1}{2} \) hypotenuse \( \sqrt{3} \)
- **45°-45°-90° triangle**
  - hypotenuse = leg \( \sqrt{2} \)
  - leg = \( \frac{1}{2} \) hypotenuse \( \sqrt{2} \)

#### Law of Sines:
\[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \] (Has an ambiguous case)

#### Law of Cosines:
\[ c^2 = a^2 + b^2 - 2ab \cos C \]

#### Area of triangle:
\[ A = \frac{1}{2} ab \sin C \]

#### Area of parallelogram:
\[ A = ab \sin C \]

#### Pythagorean Identities:
\[ \sin^2 \theta + \cos^2 \theta = 1 \]
\[ \tan^2 \theta + 1 = \sec^2 \theta \]
\[ 1 + \cot^2 \theta = \csc^2 \theta \]

#### Radians and Degrees
- Change to radians multiply by \( \frac{\pi}{180} \)
- Change to degrees multiply by \( \frac{180}{\pi} \)

#### Quadrantal angles – 0, 90, 180, 270

#### CoFunctions: examples
\[ \sin \theta = \cos(90° - \theta) \]
\[ \tan \theta = \cot(90° - \theta) \]

#### Inverse notation:
\[ \arcsin(x) = \sin^{-1}(x) \]
\[ \arccos(x) = \cos^{-1}(x) \]
\[ \arctan(x) = \tan^{-1}(x) \]

#### Reciprocal Functions
\[ \frac{1}{\sin \theta} = \csc \theta \]
\[ \frac{1}{\cos \theta} = \sec \theta \]
\[ \frac{1}{\tan \theta} = \cot \theta \]

#### Trig Functions
\[ \sin \theta = \frac{o}{h} \]
\[ \cos \theta = \frac{h}{o} \]
\[ \tan \theta = \frac{o}{a} \]
\[ \csc \theta = \frac{h}{o} \]
\[ \sec \theta = \frac{a}{h} \]
\[ \cot \theta = \frac{a}{o} \]

#### Trig Graphs
- **sin**
  - Amplitude (A) = \( \frac{1}{2} \) | max – min | (think height)
  - Period = horizontal length of 1 complete cycle
  - Frequency (B) = number of cycles in \( 2 \pi \) (period)
  - Horizontal shift (C) – movement left/right
  - Vertical shift (D) – movement up/down
Statistics and Probability – Things to Remember!

Statistics:
- **mean** = \( \bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n} \sum_{i=1}^{n} x_i \)
- **median** = middle number in ordered data
- **mode** = value occurring most often
- **range** = difference between largest and smallest

**mean absolute deviation (MAD):**
- population MAD = \( \frac{1}{n} \sum_{i=1}^{n} |x_i - \bar{x}| \)

**variance:**
- population variance = \((\sigma x)^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \)

**standard deviation:**
- population standard deviation = \( \sigma_x = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2} \)

**range**: difference between largest and smallest

**Empirical Probability**
\[ P(E) = \frac{\text{# of times event } E \text{ occurs}}{\text{total # of observed occurrences}} \]

**Theoretical Probability**
\[ P(E) = \frac{n(E)}{n(S)} = \frac{\# \text{ of outcomes in } E}{\text{total # of outcomes in } S} \]

**Binomial Probability**
- \( _n \text{C}_r \cdot p^r \cdot q^{n-r} \) “exactly” \( r \) times
- or \( \left( \begin{array}{c} n \\end{array} \right) \cdot p^r \cdot (1-p)^{n-r} \)

[TI Calculator: \text{binompdf}(n, p, r)]

When computing "at least" and "at most" probabilities, it is necessary to consider, in addition to the given probability,

- all probabilities larger than the given probability ("at least")
[TI Calculator: \text{1 - binomcdf}(n, p, r-1)]
- all probabilities smaller than the given probability ("at most")
[TI Calculator: \text{binomcdf}(n, p, r)]

**Normal Distribution and Standard Deviation**
- About 68% within 1 s.d. of mean
- About 99% within 3 s.d. of mean

- \( x \pm 2\sigma \)
- \( x \pm 3\sigma \)

**Probability Permutation:** without replacement and order matters
\[ _nP_r = \frac{n!}{(n-r)!} \]

**Combination:** without replacement and order does not matter
\[ _nC_r = \frac{n!}{r! \cdot (n-r)!} = \frac{n!}{r! (n-r)!} \]

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